Strange metals as ersatz Fermi liquids: emergent symmetries, anomalies, and experimental tests

T. Senthil (MIT)

Dominic Else, Ryan Thorngren, TS, arXiv:2007.07896.

Dominic Else and TS, arXiv: 2010.10523

Ethan Lake and TS, forthcoming

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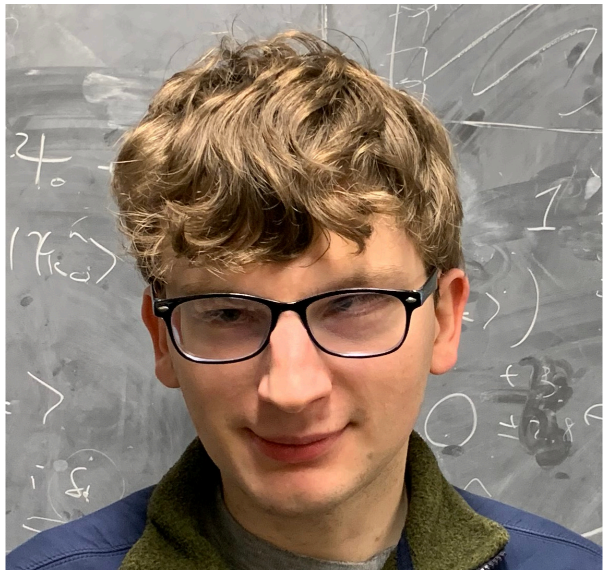
Simons Foundation

Collaborators



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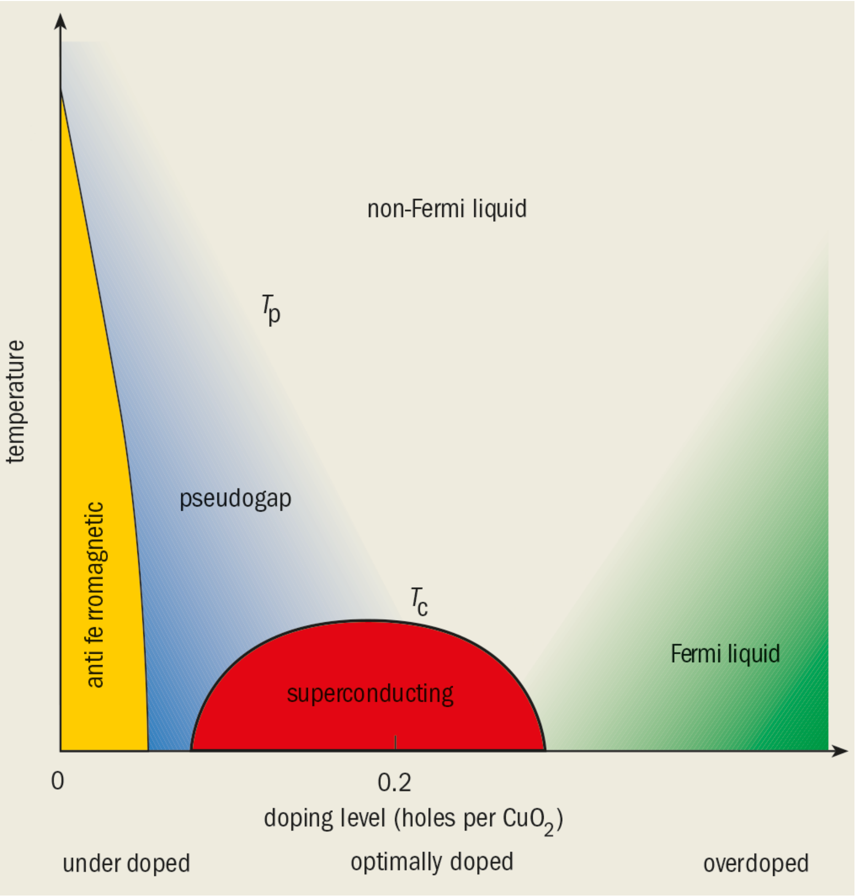
Non-fermi liquid metallic ground states

Metals that violate Fermi liquid theory down to very low temperature exist.

Prominent examples:

1. Strange metals in hole-doped cuprates

2. Heavy fermion metals near quantum critical points (eg, $YbRh_2Si_2$)



Presumably controlled by non-fermi liquid T = 0 fixed points.

Three assumptions (``central dogmas") about a class of strange metals

I. Essential physics does not involve disorder

II. Conductivity satisfies $\sigma(\omega, T) \sim \frac{1}{T}\Sigma(\frac{\omega}{T})$ at low ω,T with $\Sigma(0)$ finite.

(The famous linear dc resistivity + frequency scaling)

III. It is compressible

Why these assumptions is addressed in next few slides.

Why these assumptions?

- Well motivated by experiments on cuprates and heavy fermion QCPs

- Interesting theoretically: these assumptions are mild but still constrain theory strongly, and hence we limit the search for an explanation to a narrow range.

Three assumptions (``central dogmas") about a class of strange metals

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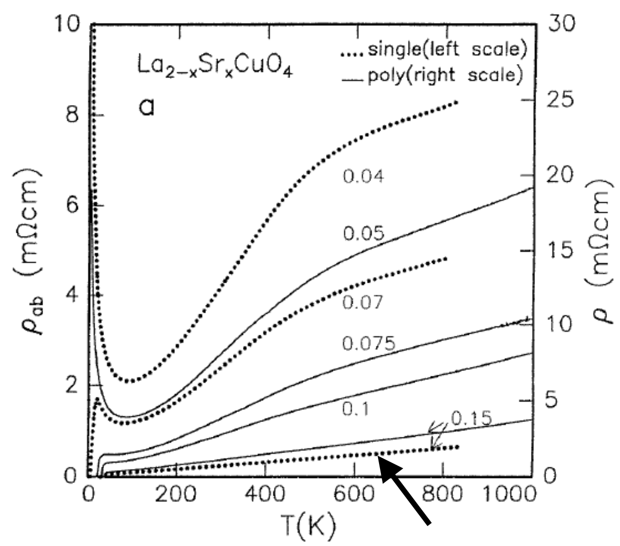
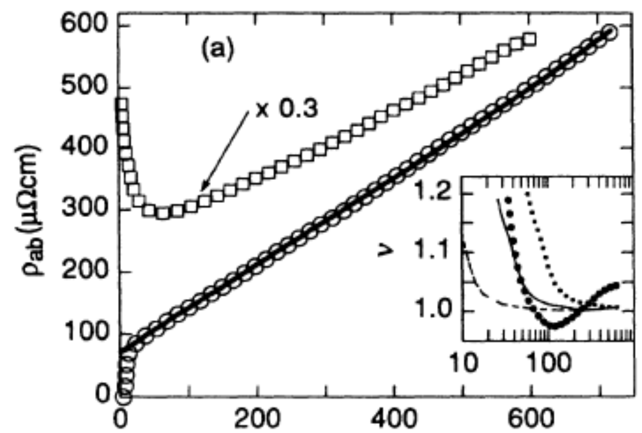
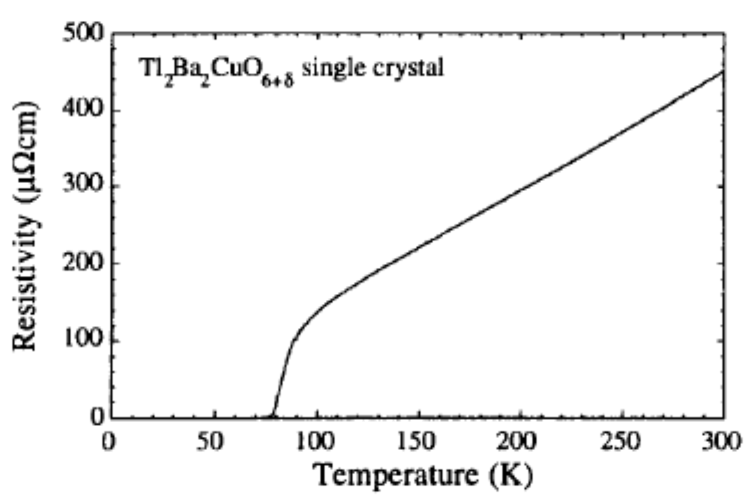
Cuprate strange metal transport



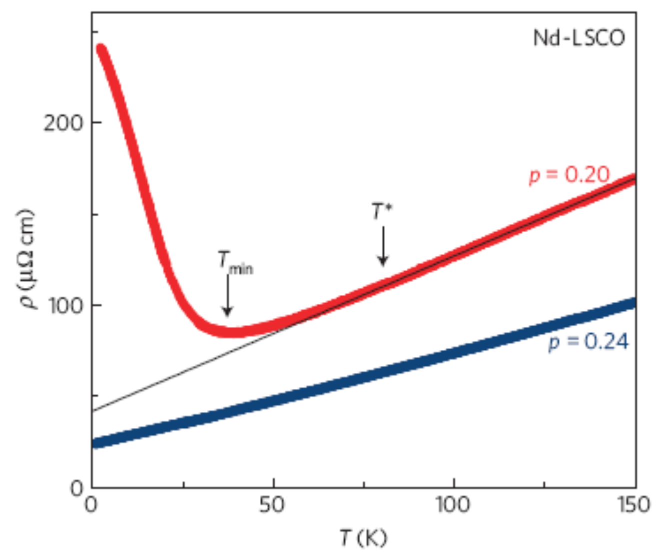
Takagi et al, 1992



Bi-2201: Martin et al, 1990



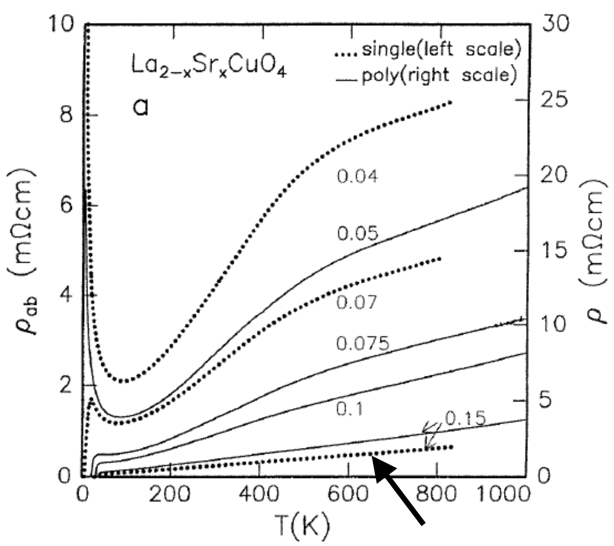
MacKenzie et al, 1997



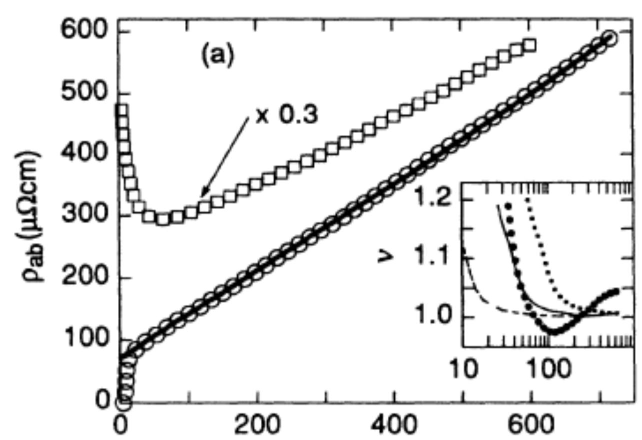
Taillefer et al, 2008

Linear T resistivity seen in diverse cuprate strange metals down to very low T.

Cuprate strange metal transport



Takagi et al, 1992

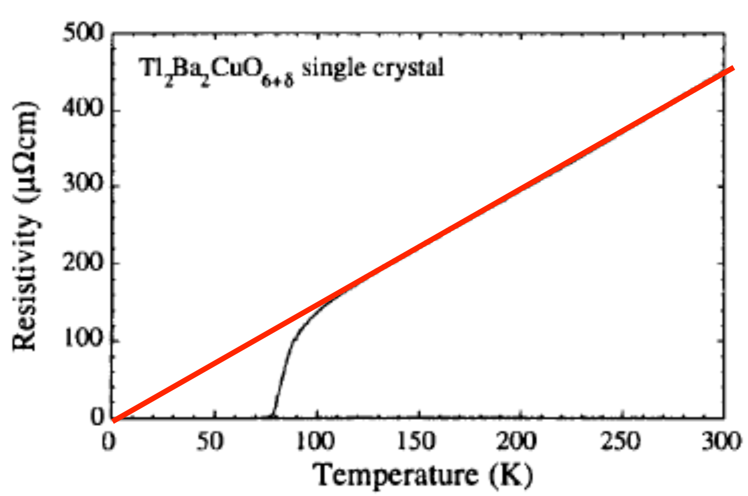


Bi-2201: Martin et al, 1990

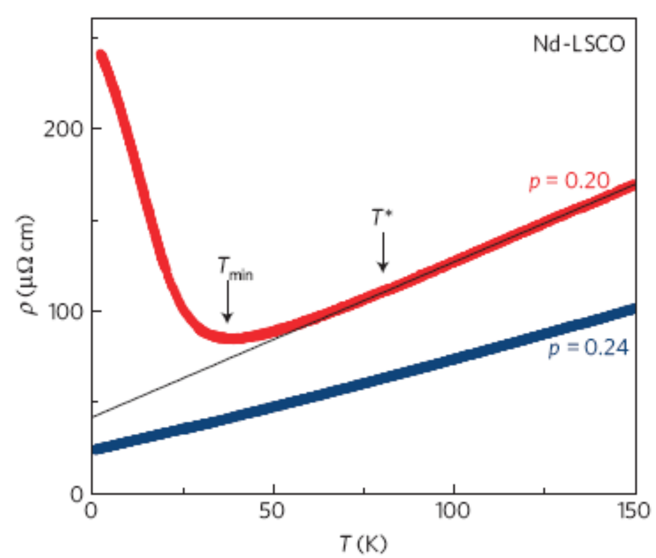
Linear T resistivity seen in diverse cuprate strange metals down to very low T.

Residual resistivity can be very small or even absent.

Perhaps disorder not essential to basic physics?



MacKenzie et al, 1997



Taillefer et al, 2008

Also, some heavy fermion quantum critical metals: stoichiometric compounds which are very clean

Cuprate strange metal and disorder

Linear (sheet) resistivity slope of each Cu-O layer is almost the same for different hole-doped cuprate materials

Table 1 | Slope of *T*-linear resistivity versus Planckian limit in seven materials

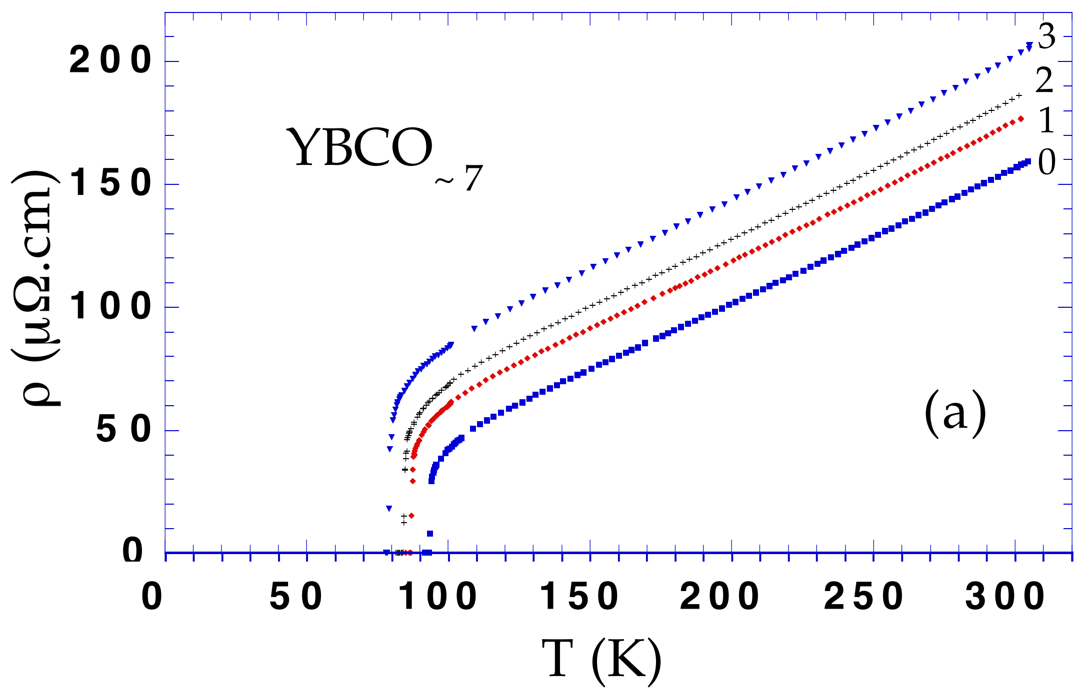
Material	Doping[a]	n (10^{27} m^{-3})	m^* (m_0)	A_1/d (Ω K^{-1})	$h/(2e^2T_F)$ (Ω K^{-1})	α
Bi2212	$p=0.23$	6.8	8.4 ± 1.6	8.0 ± 0.9	7.4 ± 1.4	1.1 ± 0.3
Bi2201	$p\sim0.4$	3.5	7 ± 1.5	8 ± 2	8 ± 2	1.0 ± 0.4
LSCO	$p=0.26$	7.8	9.8 ± 1.7	8.2 ± 1.0	8.9 ± 1.8	0.9 ± 0.3
Nd-LSCO	$p=0.24$	7.9	12 ± 4	7.4 ± 0.8	10.6 ± 3.7	0.7 ± 0.4

From Legros,……Taillefer, Nature Physics 2018

Among other differences, these will have varying disorder strengths

Cuprate strange metal and disorder (cont'd)

Studies on YBCO with electron irradiation (a gentle way to increase disorder):
Residual resistivity increases without change of slope



Rullier-Albenque, …Alloul, ..Marucco et al, 2000

Motivates assumption that essential physics of strange metal can be understood through a (lattice) translation invariant model.

Also, some heavy fermion quantum critical metals: stoichiometric compounds which are very clean

Three assumptions (``central dogmas”) about a class of strange metals

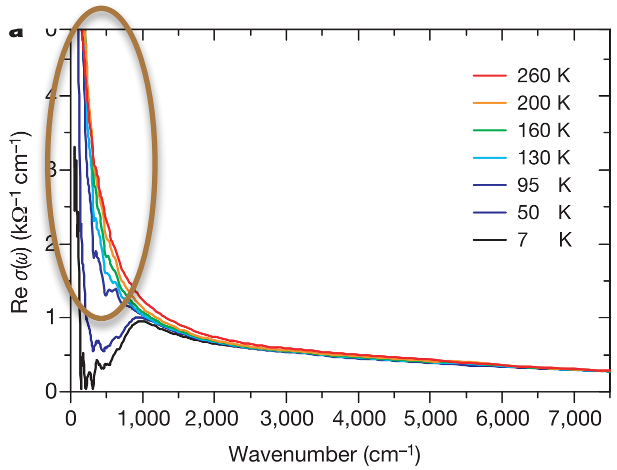
I. Essential physics does not involve disorder

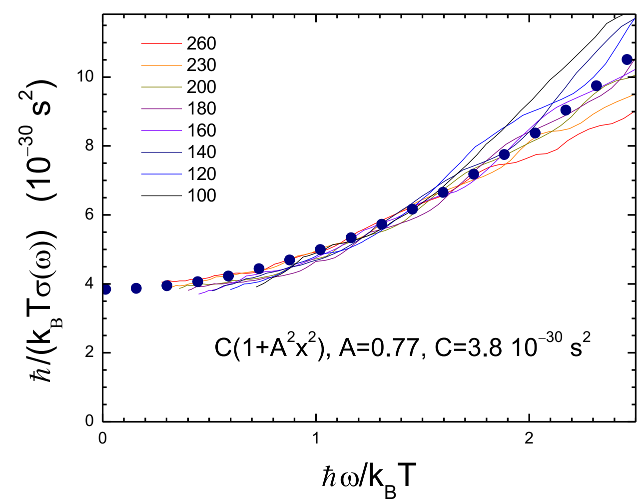
II. Conductivity satisfies $\sigma(\omega, T) \sim \frac{1}{T}\Sigma(\frac{\omega}{T})$ ***at low*** ω***, T with*** $\Sigma(0)$ ***finite.***

(The famous linear dc resistivity + frequency scaling)

III. It is compressible

Conductivity scaling

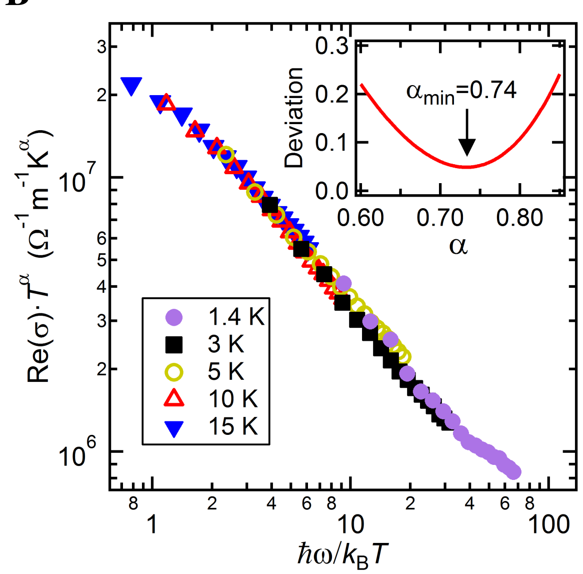




Bi2212: Van der Marel et al, 03

ω/T scaling seen upto $\omega \sim 1.5T$; (obscured at higher frequency by a power law tail upto 1 eV likely unrelated to any T = 0 quantum criticality)

B



Conductivity scaling in (quantum critical) $YbRh_2Si_2$ thin films

(Exponent α closer to 1 if data restricted to low T)

Prochaska,…..Paschen, 2020

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- Not really doubted;

Putative strange metal `quantum critical point' in cuprates

(i) occurs at different dopings in different materials

(ii) can be tuned by pressure (in one material Nd-LSCO) - see Doiron-Leiraud et al 2018

Critical doping is not pinned to any special value.

Three assumptions (``central dogmas'') about a class of strange metals

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(The famous linear dc resistivity + frequency scaling)

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I will take these for granted in rest of talk, and ask about constraints that follow on the IR theory

(i.e a T = 0 fixed point which has these properties).

Plan of talk

1. Assumptions I + III: Compressible phases in systems with lattice translation symmetry

- a modern look at Landau Fermi liquids and Luttinger's theorem

- general contraints on compressible phases

- beyond Fermi liquids: `ersatz Fermi liquids'

2. Combine with Assumption II: Transport in an ersatz Fermi liquid

- necessity of diverging susceptibility of observables with same symmetries as `loop currents'

- other experimental tests

Global symmetry in quantum many body physics

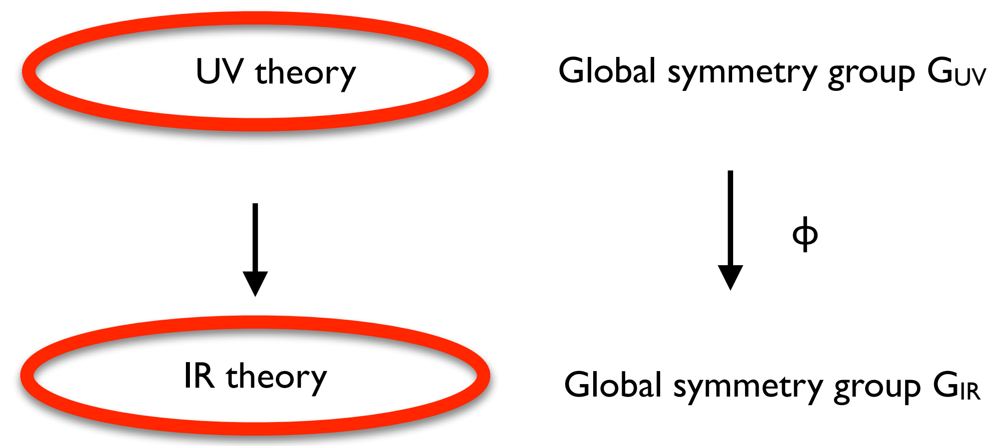
I will be interested in situations G_{UV} is an ordinary 0-form symmetry, and is not spontaneously broken in the IR.

In principle G_{IR} may include (emergent) higher form symmetries.

Global symmetry in quantum many body physics

G_{IR} may be `bigger' than G_{UV} (the IR theory may have emergent symmetry).

G_{IR} may have an t' Hooft anomaly which will be constrained by the UV theory.



Couple background gauge fields to G_{IR} .

Theory in D spacetime dimensions with t' Hooft anomaly:
Gauge invariance obtained by extending gauge field action to D+1 dimensions with a topological action related to a Symmetry Protected Topological (SPT) phase.

t'Hooft anomaly in D spacetime dimensions ↔ SPT phases in D+1 dimensions.

The UV Global symmetry

Consider UV systems with a global *internal U(1) symmetry* and *lattice translation symmetries* on a d-dimensional lattice.

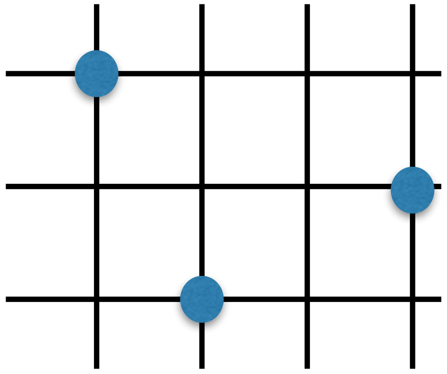
I will not specify the Hamiltonian other than to require that it is `local' (i.e is a sum of operators that each act on local regions of space).

This includes almost all models of interest in standard discussions of strongly interacting electrons (eg, the Hubbard model and variants)

Lattice filling

With a global U(1) and lattice translation symmetries, we can define the lattice filling ν = average U(1) charge per unit cell.

In a compressible phase we can tune ν continuously.

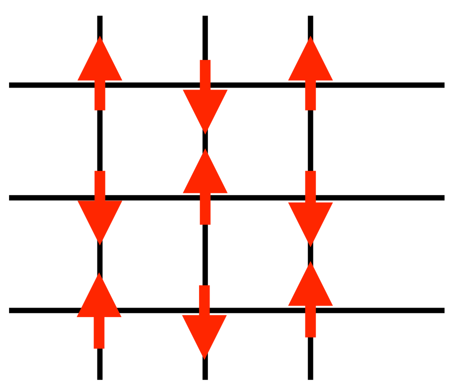


Lattice translations in the IR theory

Unit lattice translation in UV theory ~ infinitesimal translations in the IR theory

More precisely we should allow for action by an internal symmetry of the IR theory.

Example: Ising antiferromagnet



Tune to criticality $\longrightarrow$

IR - continuum ϕ^4 theory;
Unit lattice translation: $\phi \to -\phi$

There may be some exceptions to this if the IR does not involve spatial coarse-graining but we will set this subtlety aside as a future worry.

Constraints from the UV on the IR theory: Examples

0. Trivial insulators (i.e deformable to band insulators) need integer filling.

1. Lieb, Schultz, Mattis (1960s), Oshikawa (1999), Hastings (2004) theorem

For $\nu = p/q$ ground states preserving *G_{UV} = U(1) x translation* must either have topological order, or be gapless.

2. Luttinger's theorem in Fermi Liquids

Volume of Fermi surface fixed by electron filling: $$\frac{V_F}{(2\pi)^d} = \nu \ mod \ Z$$

Luttinger (1960s): perturbative proof; Oshikawa (2000): nonperturbative argument

Plan of talk

1. Assumptions I + III: Compressible phases in systems with lattice translation symmetry

<u>- a modern look at Landau Fermi liquids and Luttinger's theorem</u>

- general contraints on compressible phases

- beyond Fermi liquids: `ersatz Fermi liquids'

2. Combine with Assumption II: Transport in an ersatz Fermi liquid

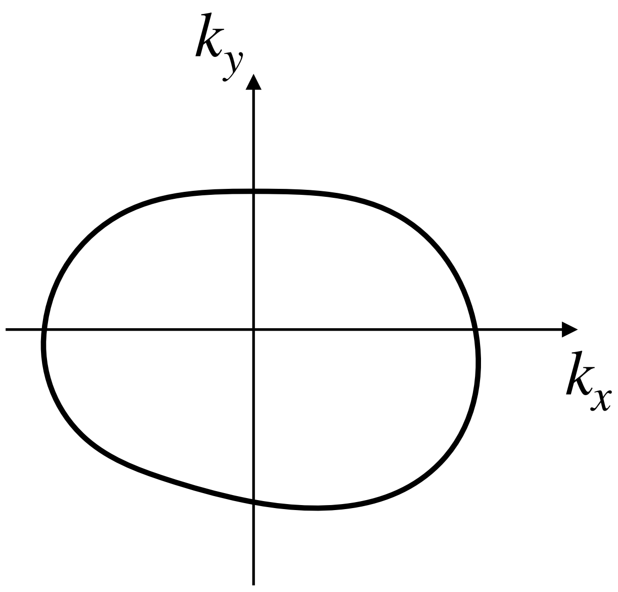
- necessity of diverging susceptibility of observables with same symmetries as `loop currents'

- other experimental tests

The Landau Fermi liquid in d = 2

G_{UV} = U(1) x lattice translations (= Z^2)

IR theory: Quasiparticles near a sharp Fermi surface



IR Hamiltonian:

$$H = \sum_k \epsilon_k n_k + 1/2 \sum_{k,k'} F_{kk'} n_k n_{k'}$$

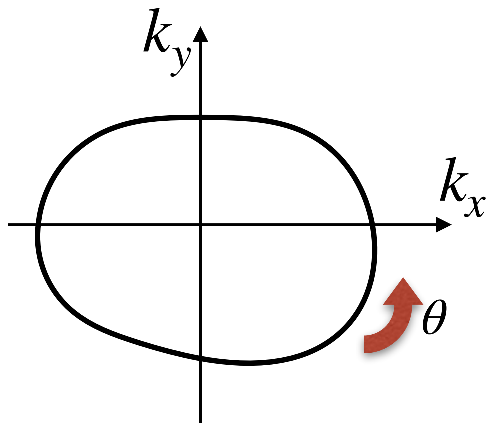
n_k = quasiparticle number at point k near Fermi surface

Emergent symmetry of the Fermi Liquid

``Quasiparticles are separately conserved for each Fermi surface point”

For each point on Fermi surface, there is a conserved charge density n_θ

$n_\theta d\theta$ is the number of quasiparticles between θ and $\theta + d\theta$



General IR symmetry element: $e^{i\int d\theta\, f(\theta)\, n_\theta}$ for smooth functions $f(\theta)$.

These define smooth maps from a circle to U(1) which form a group known as the
`loop group’ $\equiv LU(1)$ (identify as G_{IR})

Embedding microscopic symmetries

Total charge $n \sim \int d\theta \; n_\theta$

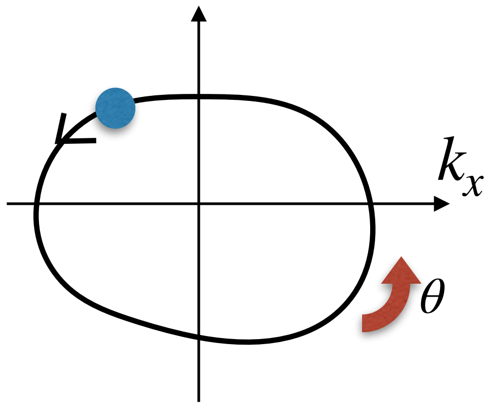
Unit lattice translations along $\alpha = (x, y)$ direction : $T_\alpha \sim e^{-i \int d\theta \; k_{F\alpha} \; n_\theta}$

(setting lattice constants to be 1).

Both U(1) and lattice translations of the UV map to elements of the LU(1) IR symmetry.

In fact we can take the action of translations to define the `Fermi momentum' in the IR theory.

t' Hooft anomaly: a physical manifestation



Turn on external electromagnetic field

Separate conservation of n_θ destroyed - only total charge is conserved.

Example: External uniform magnetic field

- quasiparticle moves around Fermi surface.

$$\frac{d\vec{k}}{dt} = -\frac{d\epsilon}{d\vec{k}} \times \vec{B}$$

Formal: study theory in presence of background gauge fields

Coupling a background gauge field to G_{IR}

t' Hooft anomaly signaled by breakdown of gauge invariance which is cured by extending the gauge fields to one higher dimension with a topological action.

For G_{IR} = loop group LU(1), the gauge field $A_\mu = (A_0, A_x, A_y, A_\theta)$ which are

all functions of of (t, x, y, θ).

Thus we have a four-dimensional gauge field.

(A_θ can be interpreted as a Berry connection on the Fermi surface).

Any anomaly will be related to a 5D topological action of a U(1) gauge field.

t' Hooft anomaly of the Fermi liquid

Corresponding topological action: 5D Chern-Simons theory

$$S[A] = \frac{m}{24\pi^2}\int A \wedge dA \wedge dA \qquad \text{with } m \in Z$$

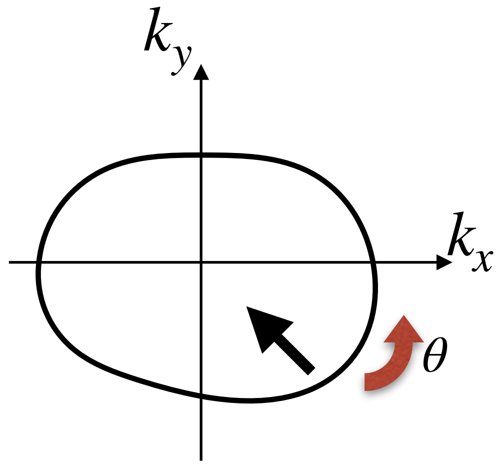
Claim: $m = \pm 1$ correctly captures physics of the Fermi liquid.

A useful physical picture

Interpret the 5th dimension as going into the interior of the Fermi surface.

We can think of the 5D U(1) gauge field as living in 2 space + 1 time + 2 momentum directions.

The Fermi surface is a boundary of the rigid occupied Fermi sea.



Other uses of `phase space' gauge fields: Bulmash, Hosur, Zhang, Qi, 2015

A manifestation of the anomaly

Turn on 2π flux of the electromagnetic field (A_x, A_y)

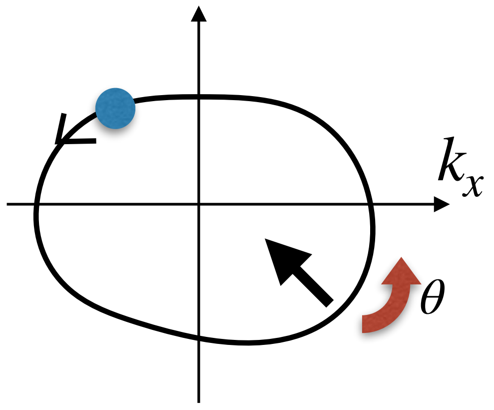
5D Chern-Simons term reduces to a 3D Chern-Simons for the remaining 3 components.

For the boundary theory at the Fermi surface the n_θ satisfy the chiral Kac-Moody algebra

$[n_\theta, n_{\theta'}] = -\frac{i}{2\pi}\delta'(\theta - \theta')$ familiar for a chiral 1d fermion

This ties in with the expectation that the quasiparticle moves chirally around Fermi surface in a magnetic field.

An interesting point of view



Turn on 2π flux of the electromagnetic field (A_x, A_y).

5D Chern-Simons term reduces to a 3D Chern-Simons for the remaining 3 components.

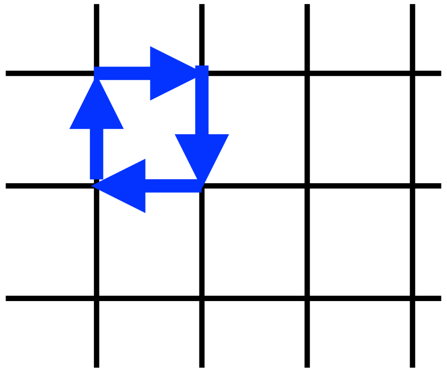
This can be thought of as an `integer quantum Hall effect' in momentum space in the interior of the Fermi surface.

The Fermi surface is the boundary of this momentum space integer

quantum Hall state, and hence has a chiral fermion.

Luttinger's theorem from the anomaly

UV theory: With 2π flux, the discrete unit translations do not commute:



$$T_x T_y T_x^{-1} T_y^{-1} = e^{2\pi i \nu}$$

IR theory: Use $T_\alpha = e^{-ia_\alpha \int d\theta\, k_{F\alpha}(\theta)\, n_\theta}$

and the commutation algebra $[n_\theta, n_{\theta'}] = -\dfrac{i}{2\pi}\delta'(\theta - \theta')$

=> $T_x T_y T_x^{-1} T_y^{-1} = e^{iV_F a_x a_y / 2\pi}$

Matching these exactly gives Luttinger's theorem.

Comments

Apart from Luttinger's theorem, several (but not all) universal properties of the Fermi liquid follow just from knowing its emergent symmetry and anomaly.

Eg: response to electric fields, quantum oscillations,….

These `kinematic' properties must be distinguished from `dynamical' properties that require knowledge of details of the IR Hamiltonian, eg, the Fermi velocity.

Plan of talk

1. Assumptions I + III: Compressible phases in systems with lattice translation symmetry

- a modern look at Landau Fermi liquids and Luttinger's theorem

- *general contraints on compressible phases*

- *beyond Fermi liquids: `ersatz Fermi liquids'*

2. Combine with Assumption II: Transport in an ersatz Fermi liquid

- necessity of diverging susceptibility of observables with same symmetries as `loop currents'

More general compressible phases

Theorem: For any irrational ν in $d > 1$ with $G_{UV} = U(1)$ x lattice translations, the emergent internal symmetry G_{IR} cannot be a compact Lie group.

Sketch (for $d = 2$) : In IR theory relate ν to anomaly of G_{IR}.

=> determined by topological action of SPT phase protected by G_{IR} in $d+1$ dimensions.

$d+1 = 3$: SPT phases with compact Lie groups are such that q copies are trivial for some finite integer q.

=> Anomaly of boundary theory is trivial for q copies of the theory, and so $\nu = p/q$ with p an integer.

Fermi liquids satisfy this as LU(1) is infinite dimensional.

Beyond Fermi liquids

Emergent ***internal symmetry*** of non-fermi liquid metals (with G_{UV} = U(1) x lattice translations):

Two possibilities

1. Infinite dimensional emergent symmetry - same as Fermi liquid or some variant thereof

``Ersatz Fermi Liquids”

Many examples (eg, standard `patch’ theory (Polchinski 94, S.-S.. Lee, 09, Metlitski and Sachdev, 10, Mross, McGreevy. Liu, TS 10) of Fermi surface + critical boson)

Beyond Fermi liquids

Emergent ***internal symmetry*** of non-fermi liquid metals (with G_{UV} = U(1) x lattice translations):

Two possibilities

1. Infinite dimensional emergent continuous symmetry - same as Fermi liquid or some variant thereof

``Ersatz Fermi Liquids”

Many examples (eg, standard `patch’ theory (Polchinski 94, S.-S.. Lee, 09, Metlitski and Sachdev, 10, Mross, McGreevy. Liu, TS 10) of Fermi surface + critical boson)

2. Finite dimensional non-compact Lie group (is this legal?)

Ersatz Fermi liquids

Even if G_{IR} and the anomaly are the same as in the fermi liquid, detailed (universal) dynamical properties can be very different.

Only `kinematic' properties will be the same.

Infinite dimensional continuous symmetry => infinite number of emergent conserved quantities.

Very strong implications for transport and other dynamical properties in such non-fermi liquid metals.

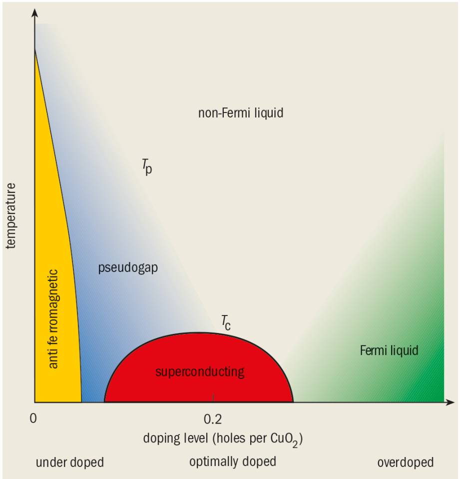
Application to experiment

I. Essential physics does not involve disorder

II. Conductivity satisfies $\sigma(\omega, T) \sim \frac{1}{T}\Sigma(\frac{\omega}{T})$ at low ω,T with $\Sigma(0)$ finite.

(The famous linear dc resistivity + frequency scaling)

III. It is compressible



So far I have only talked about consequences of Assumptions I and III.

What if we now combine with Assumption II?

Cuprate strange metal - an ersatz Fermi liquid?

Assumption II of ω/T scaling => strange metal transport must be an intrinsic property of the fixed point theory.

Tension with infinite number of emergent conserved quantities - naively mixing of current with a conserved quantity will lead to an infinite conductivity

This is familiar in systems where the IR theory conserves momentum (eg, a simple Fermi liquid), as I explain in the next slide.

IR theory with conserved momentum density $\overrightarrow{P}$

Consider equilibrium ``Hamiltonian'' density $\mathscr{H}(v) = \mathscr{H} - \overrightarrow{v} \cdot \overrightarrow{P}$

Interpret $\overrightarrow{v}$ as ``velocity''.

The electrical current density $\langle \overrightarrow{J} \rangle = \chi_{JP} \overrightarrow{v} \equiv Q \overrightarrow{v}$

(Q = charge density)

The average momentum $\langle P \rangle = \chi_{PP} \overrightarrow{v} \equiv M \overrightarrow{v}$

Uniform electric field: $\dfrac{d\langle \overrightarrow{P} \rangle}{dt} = Q \overrightarrow{E} \quad => \dfrac{d\langle \overrightarrow{J} \rangle}{dt} = \dfrac{Q^2}{M} \overrightarrow{E} = \dfrac{\chi_{JP}^2}{\chi_{PP}} \overrightarrow{E}$

Conductivity $Re\ \sigma(\omega) = \dfrac{\pi \chi_{JP}^2}{\chi_{PP}} \delta(\omega) \quad (+ \quad Re\ \sigma_{inc}(\omega))$

Transport and conservation laws

In clean Fermi liquids, the low-T conductivity is dominated by the broadening of the delta function by momentum dissipation coming from irrelevant (umklapp) operators.

More general examples :

$\overrightarrow{J}$ will overlap with all IR conserved operators with the same symmetry under G_{UV}.

These will lead to a $\delta(\omega)$ contribution to Re $\sigma(\omega)$.

A seeming paradox and its resolution

Two seemingly contradictory claims:

1. Strange metal transport is intrinsic (no delta function conductivity in fixed point theory)

2. Strange metal is ``Ersatz Fermi Liquid'' with infinite number of conserved quantities (non-zero overlap of current with some of them guaranteed by anomaly)

Resolution: Susceptibilities of all conserved quantities (that overlap with current) diverge!

Eg: In $Re\ \sigma(\omega) = \frac{\pi \chi_{JP}^2}{\chi_{PP}} \delta(\omega) \quad (+ \quad Re\ \sigma_{inc}(\omega))$, if χ_{PP} diverges, the delta function is killed.

Observables with diverging susceptibility

$\vec{J}$ will overlap with all IR conserved operators with the same symmetry under G_{UV}.

Their susceptibility must diverge at the strange metal fixed point.

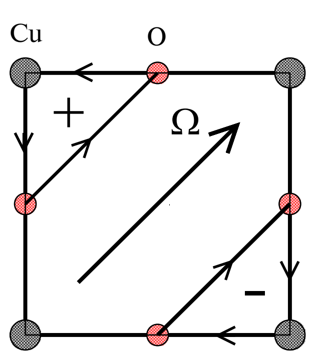
Such operators must be

- odd under time reversal, and inversion

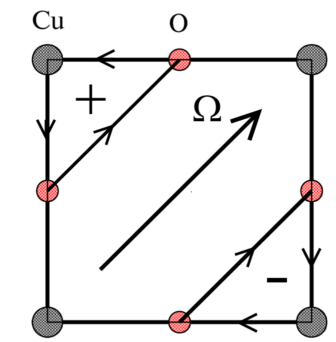
- have zero crystal momentum

- transform as a vector under lattice rotations

These are the same symmetries as the Varma loop current order !!



Loop currents and cuprates



Advocated by Chandra Varma (1990s- present)

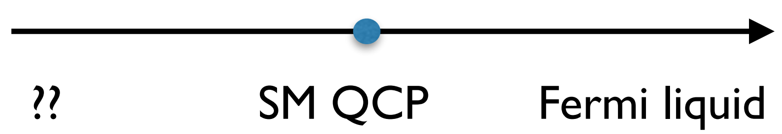
Many *reports* of static loop current order in pseudo gap regime, and many *controversies.*

Never been clear how it helps understand basic phenomenon of pseudogap (the gap opening)

We give a completely different rationale for critically fluctuating order with the same symmetries in the strange metal!

Other experimental tests

1. Unlike in a fermi liquid, strange metal transport not associated with slow relaxation of conserved quantities



Strange metal quantum critical point:
the fermi liquid T^2 resistivity will not be part of a scaling function with the strange metal resistivity.

2. Quantum oscillations: signature of emergent continuous translation symmetry in strange metal

Summary

Assuming global U(1), lattice translation symmetries, and a tunable filling lead to strong constraints on the IR theory that must be satisfied by any putative non-Fermi liquid (with these symmetries).

Ersatz Fermi liquids (with infinite dimensional emergent symmetry):

Consistent with strange metal transport only if susceptibility of certain observables with same symmetry as loop current order parameter diverges.

How far can we go without committing to detailed dynamical models?