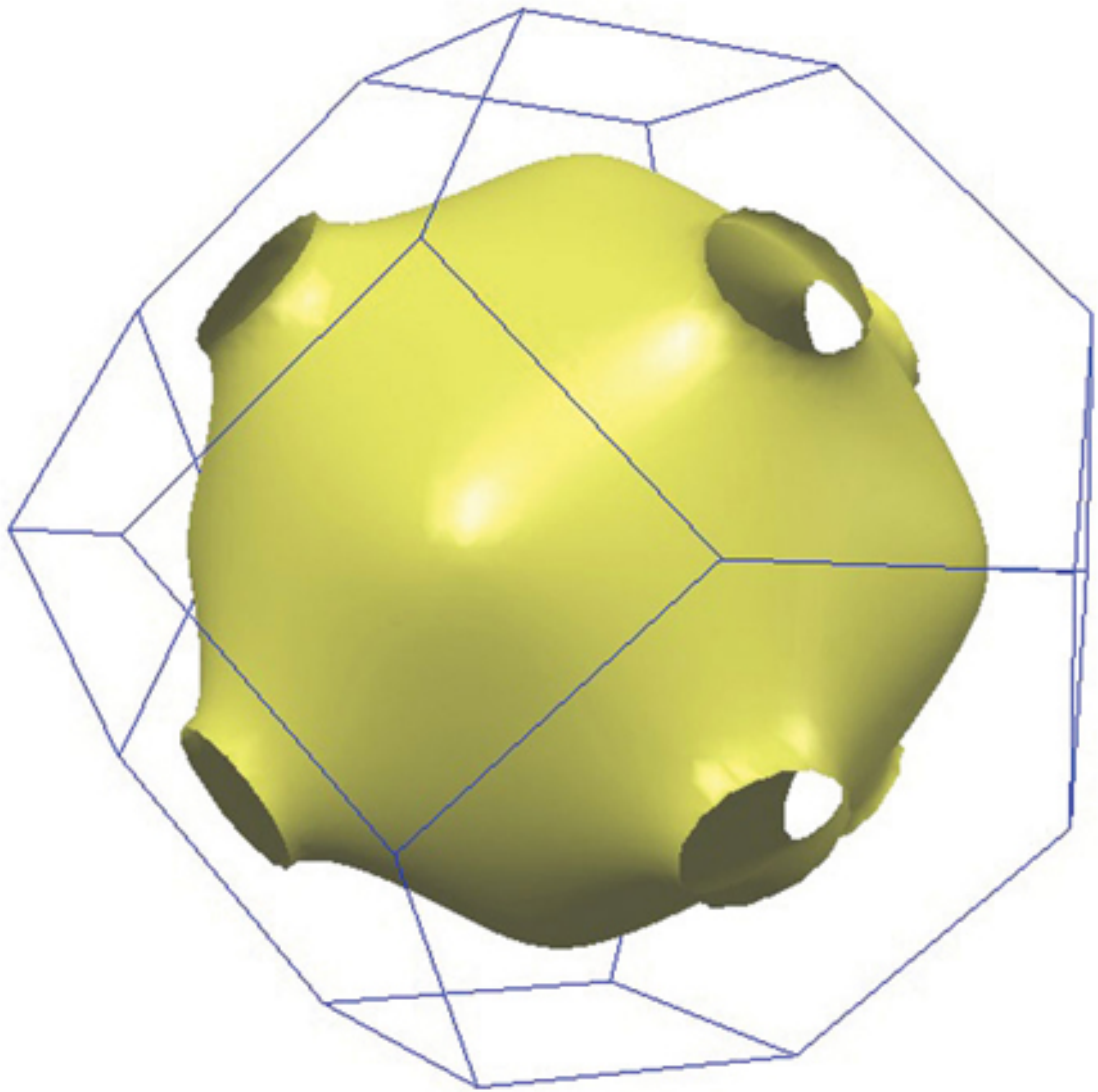

Compressible Quantum Matter: General Constraints, Emergent Symmetries, and Anomalies

T. Senthil (MIT)

Dominic Else, Ryan Thorngren, TS, arXiv:2007.07896.

Dominic Else and TS, forthcoming;
Ethan Lake and TS, forthcoming.

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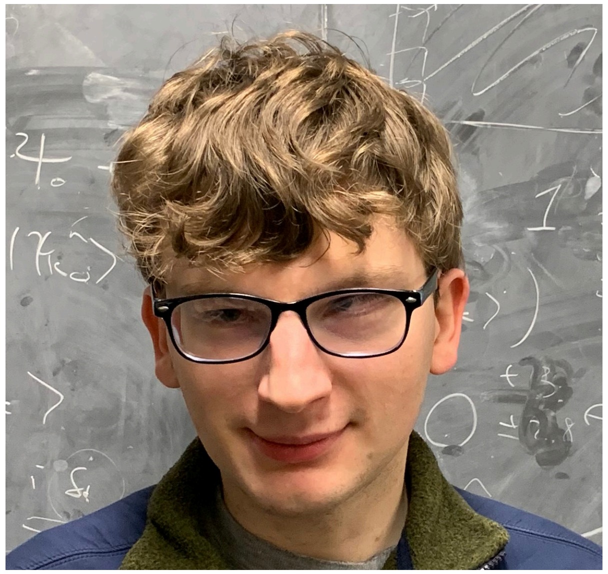
Collaborators



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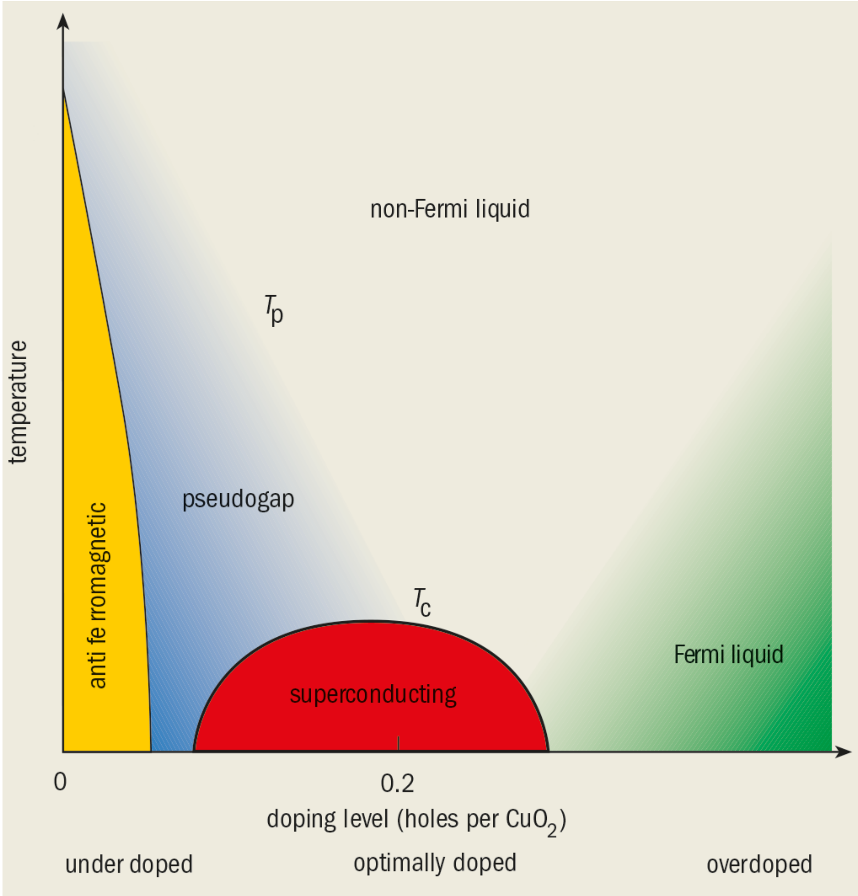
Motivation

Old challenge of how to describe ``non-fermi liquid'' metals seen in condensed matter systems.

 I will discuss very generally, in a model independent way, properties of a class of (actually IR fixed points) phases of matter.

This class will include both fermi and non-fermi liquids.



Global symmetry in quantum many body physics

$$\begin{array}{ccc} \text{UV theory} & & \text{Global symmetry group } G_{UV} \\ \downarrow & & \downarrow \phi \\ \text{IR theory} & & \text{Global symmetry group } G_{IR} \end{array}$$

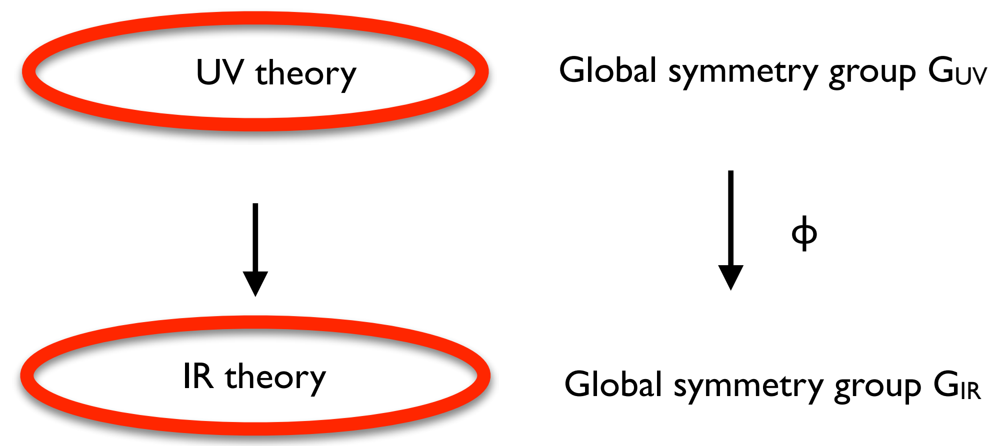
I will be interested in situations G_{UV} is an ordinary 0-form symmetry, and is not spontaneously broken in the IR.

In principle G_{IR} may include (emergent) higher form symmetries.

Global symmetry in quantum many body physics

G_{IR} may be `bigger' than G_{UV} (the IR theory may have emergent symmetry).

G_{IR} may have an t' Hooft anomaly which will be constrained by the UV theory.



Couple background gauge fields to G_{IR} .

Theory in D spacetime dimensions with t' Hooft anomaly:
Gauge invariance obtained by extending gauge field action to D+1 dimensions with a topological action related to a Symmetry Protected Topological (SPT) phase.

t'Hooft anomaly in D spacetime dimensions ↔ SPT phases in D+1 dimensions.

The UV Global symmetry

I will consider UV systems with a global *internal U(1) symmetry* and *(lattice) translation symmetries* on a d-dimensional lattice.

(In condensed matter physics the global U(1) symmetry corresponds to electric charge conservation.)

I will not specify the Hamiltonian other than to require that it is `local' (i.e is a sum of operators that each act on local regions of space).

This includes almost all models of interest in standard discussions of strongly interacting electrons (eg, the Hubbard model and variants)

Compressible quantum matter

Let n (= electrical charge) be the generator of the global U(1) symmetry, and μ the corresponding chemical potential.

The compressibility $\kappa = \frac{d\langle n \rangle}{d\mu}$.

I will be interested in phases of matter where κ is non-zero.

Within such a phase the charge density can be tuned continuously.

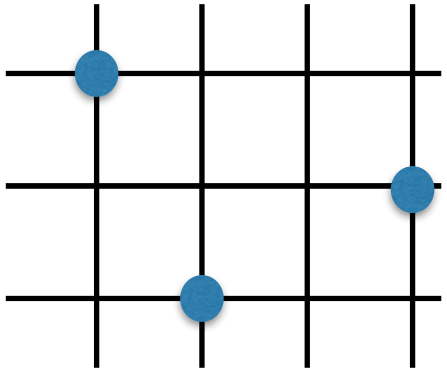
The classic example is a free fermi gas at a non-zero density.

The non-fermi liquid metals we eventually wish to understand are all compressible.

Lattice filling

With a global U(1) and lattice translation symmetries, we can define the lattice filling ν = average charge per unit cell.

In a compressible phase we can tune ν continuously.

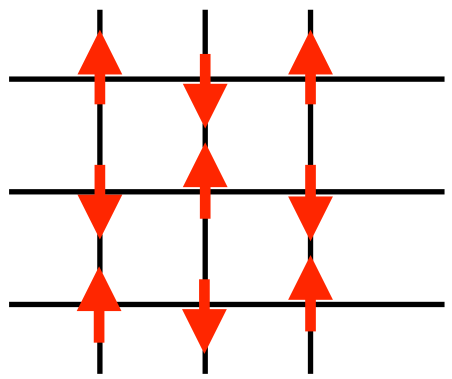


Lattice translations in the IR theory

Unit lattice translation in UV theory ~ infinitesimal translations in the IR theory

More precisely we should allow for action by an internal symmetry of the IR theory.

Example: Ising antiferromagnet



Tune to criticality $\longrightarrow$

IR - continuum ϕ^4 theory;
Unit lattice translation: $\phi \to -\phi$

There may be some exceptions to this if the IR does not involve spatial coarse-graining but we will set this subtlety aside as a future worry.

Constraints from the UV on the IR theory: a simple example

Assume IR theory is fully gapped, and is trivial (no topological order).

(Terminology in condensed matter physics: band insulator)

Only possible if UV theory has lattice filling $\nu \in Z$ (not including electron spin).

Constraints from the UV on the IR theory: More complex examples

1. Lieb, Schultz, Mattis (1960s), Oshikawa (1999), Hastings (2004) theorem

For $\nu = p/q$ ground states preserving G_{UV} = *U(1) x translation* must either have topological order, or be gapless.

2. Luttinger's theorem in Fermi Liquids

Volume of Fermi surface fixed by electron filling: $$\frac{V_F}{(2\pi)^d} = \nu \ mod \ Z$$

Luttinger (1960s): perturbative proof; Oshikawa (2000): nonperturbative argument

Plan for this talk

UV systems with $G_{UV} = U(1)$ *x translation.*

IR: assume these symmetries are unbroken, and that the ground state is compressible (ν can be continuously tuned).

We will see that there are strong constraints - which generalize Luttinger's theorem - on the structure of the IR theory.

These constraints must be satisfied by any IR model that is emergeable (i.e capable of emerging) from the UV theory with the specified symmetries.

Along the way we will obtain a new non-perturbative proof of Luttinger's theorem.

(More detailed) plan of talk

A. d = 1 : Chiral anomaly and Luttinger's theorem

B. d = 2 Fermi liquids

- Emergent `loop group' symmetry
- Relevance of the 5D Chern-Simons action
- Luttinger's theorem from the anomaly.

C. General theorem for compressible phases in d > 1

- Non-fermi liquids as `Ersatz Fermi liquids'

D. (Time permitting) Application to experiments

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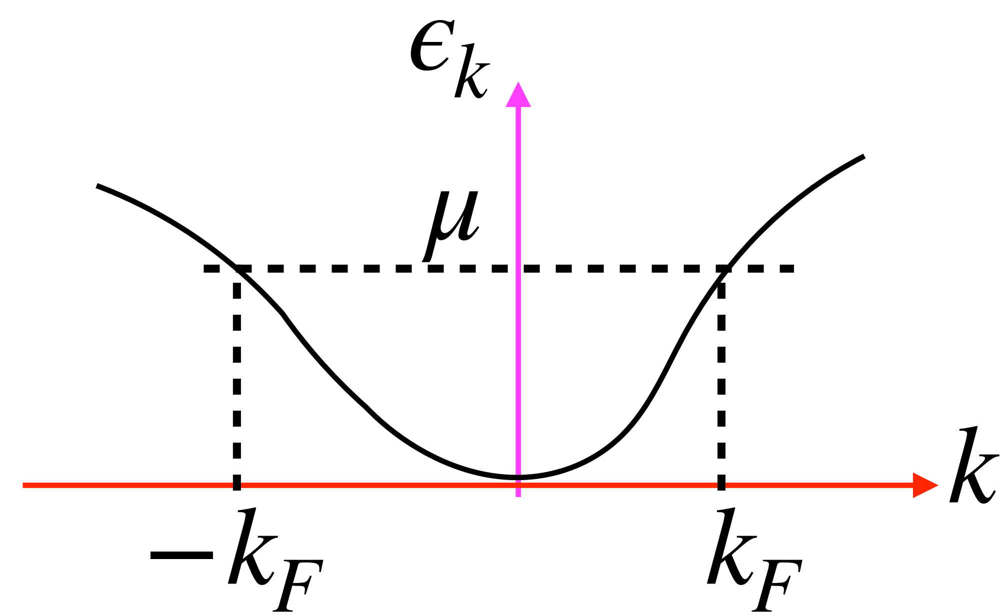
D. (Time permitting) Application to experiments

1d compressible matter

Free fermions at non-zero density in1d:

IR theory - massless Dirac fermion



Global symmetry U(1) x U(1)

Add interactions: marginal perturbation leading to a fixed line
(condensed matter physics: a.k.a Luttinger Liquid)

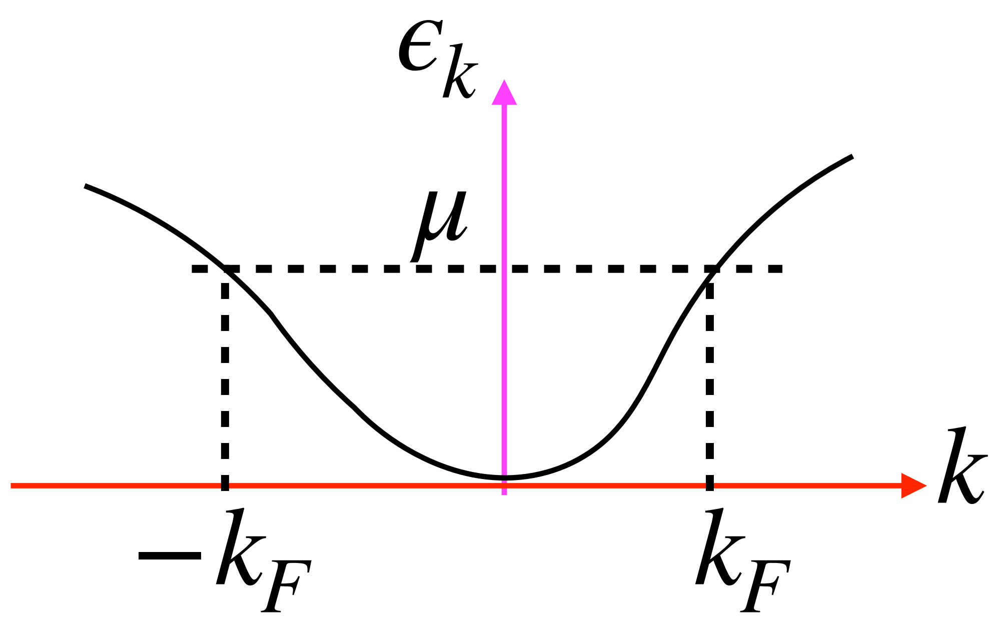
Preserves U(1) x U(1) symmetry.

1d compressible matter (cont'd)

Total charge $Q \sim n_L + n_R$

Total momentum(*) $P \sim k_F(n_R - n_L)$

(Embedding the G_{UV} into G_{IR})



IR global symmetry U(1) x U(1) is broken by external gauge fields, eg, turn on electric field E coupling to total charge.

$$\partial_\mu j^\mu_L = -E/2\pi$$

Chiral anomaly (example of t' Hooft anomaly)

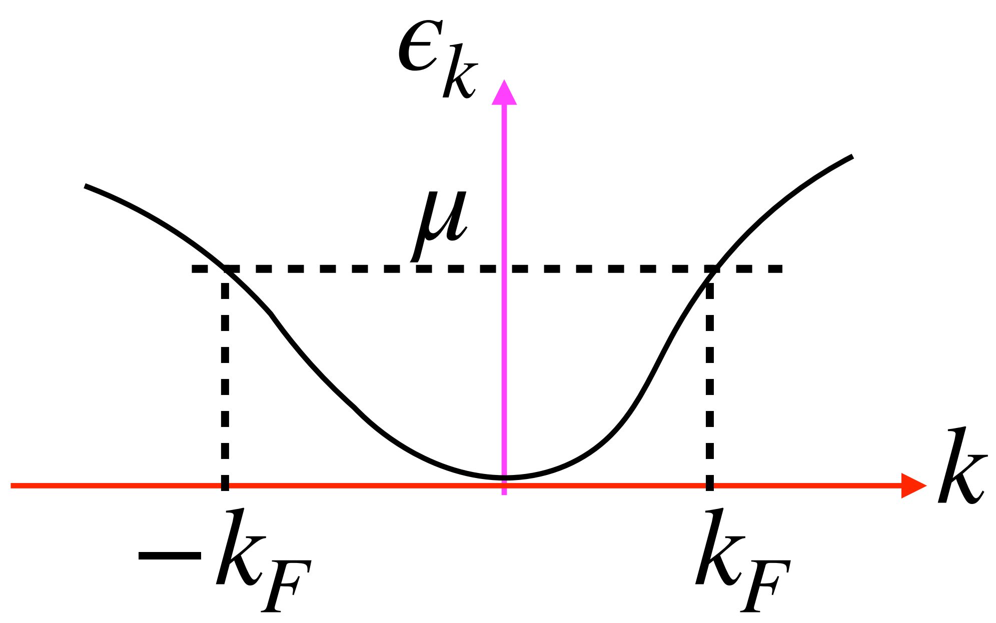
$$\partial_\mu j^\mu_R = E/2\pi$$

(*) For simplicity, assume continues translation symmetry in UV; argument can be extended if there is a lattice.

Chiral anomaly and Luttinger's theorem

Total charge $Q \sim n_L + n_R$

Total momentum $P \sim k_F(n_R - n_L)$



In original UV theory: $dP/dt = nE$

In IR theory: (from anomaly) $dP/dt = k_F d(n_R - n_L)/dt = k_F EL/\pi$
(L = length of system)

Comparing gives $k_F = \pi n/L$ which is Luttinger's theorem

(More detailed) plan of talk

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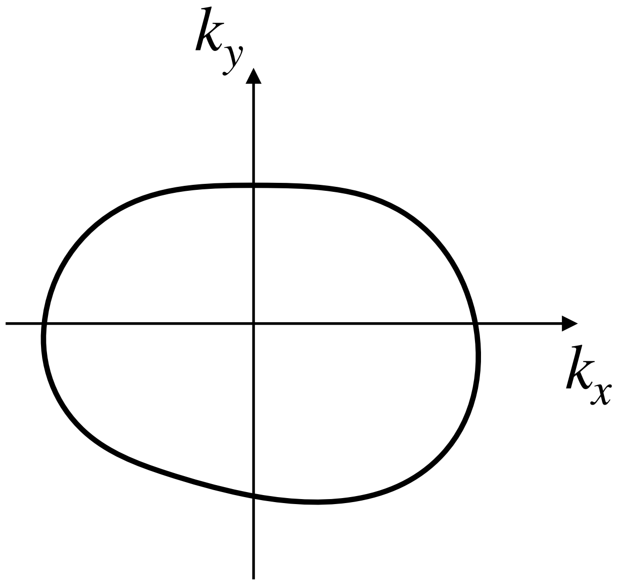
D. (Time permitting) Application to experiments

The Landau Fermi liquid in d = 2

G_{UV} = U(1) x lattice translations (= Z^2)

IR theory: Quasiparticles near a sharp Fermi surface



IR Hamiltonian:

$$H = \sum_k \epsilon_k n_k + 1/2 \sum_{k,k'} F_{kk'} n_k n_{k'}$$

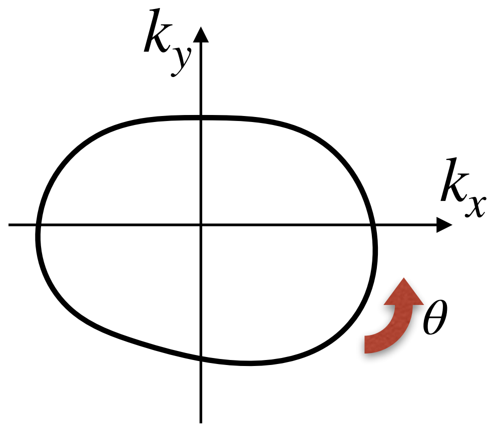
n_k = quasiparticle number at point k near Fermi surface

Emergent symmetry of the Fermi Liquid

``Quasiparticles are separately conserved for each Fermi surface point"

For each point on Fermi surface, there is a conserved charge density n_θ

$n_\theta d\theta$ is the number of quasiparticles between θ and $\theta + d\theta$



General IR symmetry element: $e^{i \int d\theta \; f(\theta) \; n_\theta}$ for smooth functions $f(\theta)$.

These define smooth maps from a circle to U(1) which form a group known as the
`loop group' $\equiv LU(1)$ (identify as G_{IR})

Embedding microscopic symmetries

Total charge $n \sim \int d\theta \; n_\theta$

Unit lattice translations along $\alpha = (x, y)$ direction : $T_\alpha \sim e^{-i \int d\theta \; k_{F\alpha} \; n_\theta}$

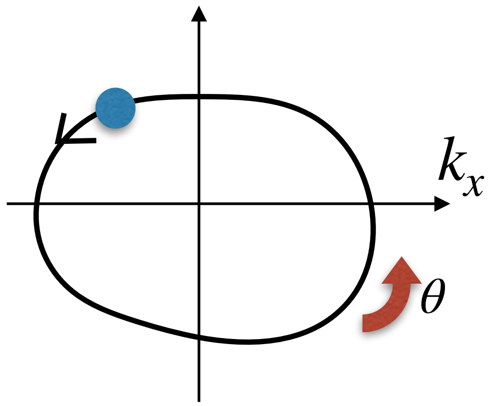
(setting lattice constants to be 1).

Both U(1) and lattice translations of the UV map to elements of the LU(1) IR symmetry.

In fact we can take the action of translations to define the `Fermi momentum' in the IR theory.

t' Hooft anomaly: a physical manifestation

Turn on external electromagnetic field



Separate conservation of n_θ destroyed - only total charge is conserved.

Example: External uniform magnetic field

- quasiparticle moves around Fermi surface.

$$\frac{d\vec{k}}{dt} = -\frac{d\epsilon}{d\vec{k}} \times \vec{B}$$

Formal: study theory in presence of background gauge fields

Coupling a background gauge field to G_{IR}

t' Hooft anomaly signaled by breakdown of gauge invariance which is cured by extending the gauge fields to one higher dimension with a topological action.

For G_{IR} = loop group LU(1), the gauge field $A_\mu = (A_0, A_x, A_y, A_\theta)$ which are

all functions of of (t, x, y, θ).

Thus we have a four-dimensional gauge field.

(A_θ can be interpreted as a Berry connection on the Fermi surface).

Any anomaly will be related to a 5D topological action of a U(1) gauge field.

t' Hooft anomaly of the Fermi liquid

Corresponding topological action: 5D Chern-Simons theory

$$S[A] = \frac{m}{24\pi^2}\int A \wedge dA \wedge dA \qquad \text{with } m \in Z$$

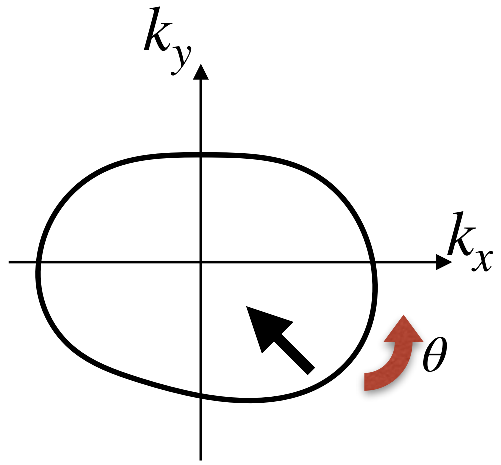
Claim: $m = \pm 1$ correctly captures physics of the Fermi liquid.

A useful physical picture

Interpret the 5th dimension as going into the interior of the Fermi surface.

We can think of the 5D U(1) gauge field as living in 2 space + 1 time + 2 momentum directions.

The Fermi surface is a boundary of the rigid occupied Fermi sea.



A manifestation of the anomaly

Turn on 2π flux of the electromagnetic field (A_x, A_y)

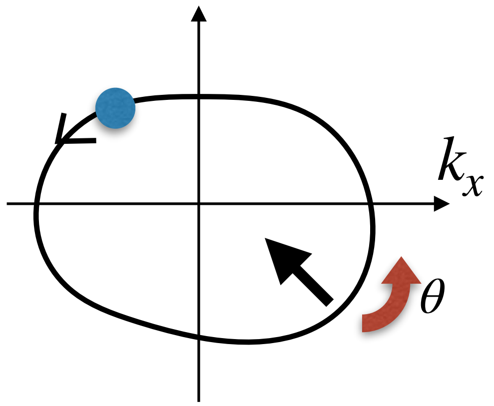
5D Chern-Simons term reduces to a 3D Chern-Simons for the remaining 3 components.

For the boundary theory at the Fermi surface the n_θ satisfy the chiral Kacs-Moody algebra

$[n_\theta, n_{\theta'}] = -\frac{i}{2\pi}\delta'(\theta - \theta')$ familiar for a chiral 1d fermion

This ties in with the expectation that the quasiparticle moves chirally around Fermi surface in a magnetic field.

An interesting point of view



Turn on 2π flux of the electromagnetic field (A_x, A_y).

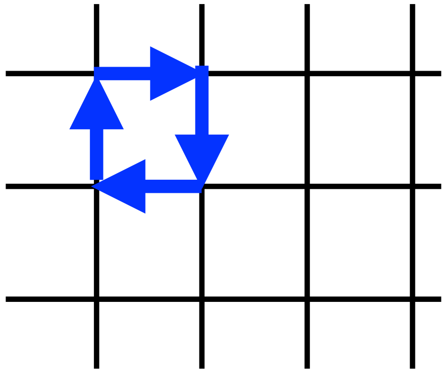
5D Chern-Simons term reduces to a 3D Chern-Simons for the remaining 3 components.

This can be thought of as an `integer quantum Hall effect' in momentum space in the interior of the Fermi surface.

The Fermi surface is the boundary of this momentum space integer

quantum Hall state, and hence has a chiral fermion.

Luttinger's theorem from the anomaly



UV theory: With 2π flux, the discrete unit translations do not commute:

$$T_x T_y T_x^{-1} T_y^{-1} = e^{2\pi i \nu}$$

IR theory: Use $T_\alpha = e^{-ia_\alpha \int d\theta \; k_{F\alpha}(\theta) \; n_\theta}$

and the commutation algebra $[n_\theta, n_{\theta'}] = -\dfrac{i}{2\pi}\delta'(\theta - \theta')$

=> $T_x T_y T_x^{-1} T_y^{-1} = e^{iV_F a_x a_y / 2\pi}$

Matching these exactly gives Luttinger's theorem.

Comments

Apart from Luttinger's theorem, several (but not all) universal properties of the Fermi liquid follow just from knowing its emergent symmetry and anomaly.

Eg: response to electric fields, quantum oscillations,….

These `kinematic' properties must be distinguished from `dynamical' properties that require knowledge of details of the IR Hamiltonian, eg, the Fermi velocity.

More general compressible phases

Theorem: For any irrational ν in $d > 1$ with $G_{UV} = U(1)$ x lattice translations, G_{IR} cannot be a compact Lie group.

Can prove so long as G_{IR} only includes 0-form symmetries (conjecture: it holds even with higher form symmetries).

Sketch (for $d = 2$) : In IR theory relate ν to anomaly of G_{IR}.

=> determined by topological action of SPT phase protected by G_{IR} in $d+1$ dimensions.

$d+1 = 3$: SPT phases with compact Lie groups are such that q copies are trivial for some finite integer q.

=> Anomaly of boundary theory is trivial for q copies of the theory, and so $\nu = p/q$ with p an integer.

Fermi liquids satisfy this as $LU(1)$ is infinite dimensional.

Beyond Fermi liquids

Emergent internal symmetry of non-fermi liquid metals (with G_{UV} = U(1) x lattice translations):

Two possibilities

1. Infinite dimensional emergent symmetry - same as Fermi liquid or some variant thereof

``Ersatz Fermi Liquids"

Many examples (eg, standard `patch' theory (Polchinski 94, S.-S.. Lee, 09, Metlitski and Sachdev, 10, Mross, McGreevy. Liu, TS 10) of Fermi surface + critical boson)

Beyond Fermi liquids

Emergent internal symmetry of non-fermi liquid metals (with G_{UV} = U(1) x lattice translations):

Two possibilities

1. Infinite dimensional emergent continuous symmetry - same as Fermi liquid or some variant thereof

``Ersatz Fermi Liquids”

Many examples (eg, standard `patch’ theory (Polchinski 94, S.-S.. Lee, 09, Metlitski and Sachdev, 10, Mross, McGreevy. Liu, TS 10) of Fermi surface + critical boson)

2. Finite dimensional non-compact Lie group (is this legal?)

Ersatz Fermi liquids

Even if G_{IR} and the anomaly are the same as in the fermi liquid, detailed (universal) dynamical properties can be very different.

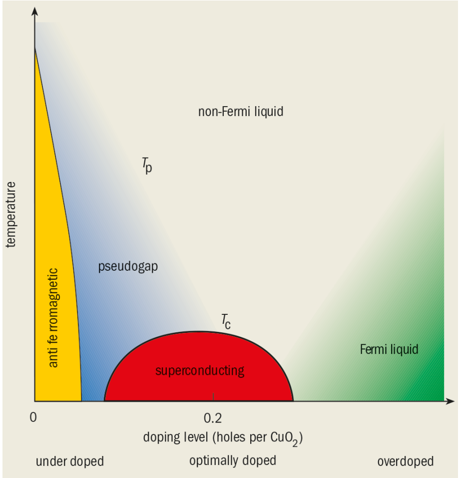
Only `kinematic' properties will be the same.

Infinite dimensional continuous symmetry => infinite number of emergent conserved quantities.

Very strong implications for transport and other dynamical properties in such non-fermi liquid metals.

Application to experiments

(Dominic Else and TS, forthcoming)



Could the cuprate strange metal be an ersatz Fermi liquid?

Some assumptions about the cuprate strange metal:

1. Essential physics does not involve disorder (G_{UV} = U(1) x lattice translations)

(Some support from experiment but controversial)

2. It is compressible

(No reason to doubt)

3. Conductivity satisfies $\sigma(\omega, T) \sim \frac{1}{T}\Sigma(\frac{\omega}{T})$ at low ω,T.

(dc limit: famous linear resistivity; some experimental support for scaling with frequency).

4. Controlled by a T = 0 fixed point with zero residual entropy

(Hinted by experiments; residual entropy if present is very small (Lake and TS, to appear); note that holographic/ SYK NFLs all have residual entropy).

Cuprate strange metal - an ersatz Fermi liquid?

With these assumptions, we can argue that the transport must be an intrinsic property of the fixed point theory.

Tension with infinite number of emergent conserved quantities - naively mixing of current with a conserved quantity will lead to an infinite conductivity (as in a Fermi liquid).

To get a finite conductivity, it is necessary that certain susceptibilities of these conserved quantities diverge.

A prediction: Strange metal, if an Ersatz Fermi Liquid, will have divergent susceptibility of inversion/time reversal breaking orders at zero crystal momentum (eg, fluctuating loop current order)

Summary and outlook

Assuming global U(1), lattice translation symmetries, and a tunable filling lead to strong constraints on the IR theory that must be satisfied by any putative non-Fermi liquid (with these symmetries).

Ersatz Fermi liquids (with infinite dimensional emergent symmetry): are they the only possibility?

How far can we go without committing to detailed dynamical models?